JEE MAIN Sept. 2020

QUESTION PAPER WITH SOLUTION

MATHEMATICS _ 6 Sep. _ SHIFT - 1

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Q.1 The region represented by $\{z = x + iy \in C : |z| - Re(z) \le 1\}$ is also given by the inequality:
$\{z = x + iy \in C : |z| - Re(z) \le 1\}$
$\{z = x + iy \in C : |z| - Re(z) \le 1\}$ के द्वारा निरूपित क्षेत्र, निम्न असमिका द्वारा दिया गया है–

(1) $y^2 \le 2\left(x + \frac{1}{2}\right)$ (2) $y^2 \le x + \frac{1}{2}$ (3) $y^2 \ge 2(x + 1)$ (4) $y^2 \ge x + 1$

Sol. **1**
$\{z = x + iy \in c : |z| - Re(z) \le 1\}$
$|z| = \sqrt{x^2 + y^2}$
$Re(z) = x$
$|z| - Re(z) \le 1$
$\Rightarrow \sqrt{x^2 + y^2} - x \le 1$
$\Rightarrow \sqrt{x^2 + y^2} \le 1 + x$
$\Rightarrow x^2 + y^2 \le 1 + x^2 + 2x$
$\Rightarrow y^2 \le 2\left(x + \frac{1}{2}\right)$

Q.2 The negation of the Boolean expression $p \vee (\sim p \wedge q)$ is equivalent to:
बूलियन व्यंजक $p \vee (\sim p \wedge q)$ की निषेधन (negation) निम्न के बराबर है–
(1) $p \wedge \sim q$ (2) $\sim p \vee \sim q$ (3) $\sim p \vee q$ (4) $\sim p \wedge \sim q$

Sol. **4**
$p \vee (\sim p \wedge q)$
$(p \wedge \sim p) \wedge (p \vee q)$
$t \wedge (p \vee q)$
$p \vee q$
$\sim (p \vee (\sim p \wedge q)) = \sim (p \vee q)$
$= (\sim p) \wedge (\sim q)$

Q.3 The general solution of the differential equation $\sqrt{1+x^2+y^2+x^2y^2} + xy\frac{dy}{dx} = 0$ is:
(where C is a constant of integration)

अवकल समीकरण $\sqrt{1+x^2+y^2+x^2y^2} + xy\frac{dy}{dx} = 0$ का व्यापक हल होगा :
(जहां C समाकलन का एक नियतांक है)

(1) $\sqrt{1+y^2} + \sqrt{1+x^2} = \frac{1}{2}\log_e\left(\frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1}\right) + C$

(2) $\sqrt{1+y^2} - \sqrt{1+x^2} = \frac{1}{2}\log_e\left(\frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1}\right) + C$

(3) $\sqrt{1+y^2} + \sqrt{1+x^2} = \frac{1}{2}\log_e\left(\frac{\sqrt{1+x^2}+1}{\sqrt{1+x^2}-1}\right) + C$

(4) $\sqrt{1+y^2} - \sqrt{1+x^2} = \frac{1}{2}\log_e\left(\frac{\sqrt{1+x^2}+1}{\sqrt{1+x^2}-1}\right) + C$

Sol. **3**

$$\sqrt{1+x^2+y^2+x^2y^2} + xy\frac{dy}{dx} = 0$$

$$\sqrt{(1+x^2)(1+y^2)} + xy\frac{dy}{dx} = 0$$

$$\frac{\sqrt{(1+x^2)}dx}{x} = -\frac{y}{\sqrt{1+y^2}}dy$$

Integrate the equation

$$\int\frac{\sqrt{1+x^2}}{x}dx = -\int\frac{y}{\sqrt{1+y^2}}dy$$

$1 + x^2 = t^2$ $\qquad$ $1 + y^2 = z^2$

$2xdx = 2tdt$

$dx = \frac{t}{x}dt$ $\qquad$ $2ydy = 2zdz$

$$\int\frac{t.tdt}{t^2-1} = -\int\frac{zdx}{z}$$

$$\int \frac{t^2-1+1}{t^2-1}dt = -z+c$$

$$\int 1dt + \int \frac{1}{t^2-1}dt = -z+c$$

$$t + \frac{1}{2}\ln\left(\frac{t-1}{t+1}\right) = -z+c$$

$$\sqrt{1+x^2} + \frac{1}{2}\ln\left(\frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1}\right) = -\sqrt{1+y^2}+c$$

$$\sqrt{1+y^2} + \sqrt{1+x^2} = \frac{1}{2}\ln\left(\frac{\sqrt{x^2+1}+1}{\sqrt{x^2+1}-1}\right) + c$$

Q.4 Let L_1 be a tangent to the parabola $y^2 = 4(x + 1)$ and L_2 be a tangent to the parabola $y^2 = 8(x + 2)$ such that L_1 and L_2 intersect at right angles. Then L_1 and L_2 meet on the straight line:

माना L_1 परवलय $y^2 = 4(x + 1)$ की एक स्पर्श रेखा है, तथा L_2 परवलय $y^2 = 8(x + 2)$ की एक स्पर्शरेखा है ताकि L_1 तथा L_2 समकोण पर प्रतिच्छेद करती है। तब L_1 तथा L_2 निम्न सरल रेखा पर मिलती है।

(1) $x + 2y = 0$ (2) $x + 2 = 0$ (3) $2x + 1 = 0$ (4) $x + 3 = 0$

Sol. **4**

Let t_1 tangent of $y^2 = 4(x + 1)$

$L_1 : t_1y = (x + 1) + t_1^2$ ....(i)

and t_2 tangent of $y^2 = 8 (x + 2)$

$L_2 : t_2y = (x + 2) + 2 t_2^2$

$L_1 \perp L_2$

$$\frac{1}{t_1}\cdot\frac{1}{t_2} = -1$$

$t_1t_2 = -1$

$t_2(i) - t_1$ (ii)

$t_1t_2y = t_2 (x + 1) + t_2. t_1^2$

$t_1t_2y = t_1 (x + 2) + 2t_2^2. t_1$

$-\quad\quad -\quad\quad\quad -$

$(t_2-t_1) x + (t_2-2t_1) + t_2t_1(t_1-2t_2) = 0$

$(t_2 - t_1) x + (t_2 - 2t_1) - (t_1 - 2t_2) = 0$

$(t_2 - t_1) x + 3t_2 - 3t_1 = 0$

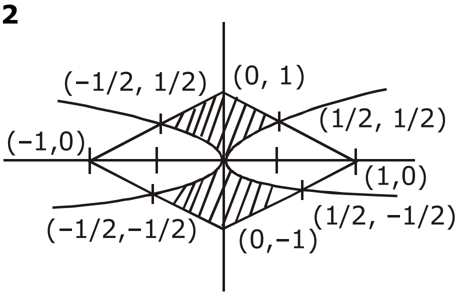
$\Rightarrow x + 3 = 0$

Q.5 The area (in sq. units) of the region A = {(x, y): |x| + |y| $\le$ 1, $2y^2 \ge |x|$}

क्षेत्र A = {(x, y): |x| + |y| $\le$ 1, $2y^2 \ge |x|$} का क्षेत्रफल (वर्ग इकाई में) होगा–

(1) $\frac{1}{6}$ (2) $\frac{5}{6}$ (3) $\frac{1}{3}$ (4) $\frac{7}{6}$

Sol. **2**



Total area = $4\int_0^{1/2}\left[(1-x)-\left(\sqrt{\frac{x}{2}}\right)\right]dx$

$= 4\left[x-\frac{x^2}{2}-\frac{1}{\sqrt{2}}\frac{x^{3/2}}{3/2}\Big|_0^{1/2}\right]$

$= 4\left[\frac{1}{2}-\frac{1}{8}-\frac{\sqrt{2}}{3}\left(\frac{1}{2}\right)^{3/2}\right]$

$= 4\times\frac{5}{24}=\frac{5}{6}$

Q.6 The shortest distance between the lines $\frac{x-1}{0}=\frac{y+1}{-1}=\frac{z}{1}$ and x + y + z + 1 = 0, 2x – y + z + 3 = 0 is:

रेखा $\frac{x-1}{0}=\frac{y+1}{-1}=\frac{z}{1}$ तथा x + y + z + 1 = 0, 2x – y + z + 3 = 0 के बीच न्यूनतम दूरी है–

(1) 1 (2) $\frac{1}{\sqrt{2}}$ (3) $\frac{1}{\sqrt{3}}$ (4) $\frac{1}{2}$

Sol. **3**

Plane through line of intersection is

$x + y + z + 1 + \lambda(2x - y + z + 3) = 0$

It should be parallel to given line

$0(1 + 2\lambda) - 1(1 - \lambda) + 1(1 + \lambda) = 0 \Rightarrow \lambda = 0$

Plane: x + y + z + 1 = 0

Shortest distance of (1, –1, 0) from this plane

$= \frac{|1-1+0+1|}{\sqrt{1^2+1^2+1^2}}=\frac{1}{\sqrt{3}}$

Q.7 Let a, b, c, d and p be any non zero distinct real numbers such that $(a^2 + b^2 + c^2)p^2 - 2(ab + bc + cd)p + (b^2 + c^2 + d^2) = 0$. Then:

(1) a, c, p are in G.P. (2) a, b, c, d are in G.P.
(3) a, b, c, d are in A.P. (4) a, c, p are in A.P.

माना a, b, c, d तथा p कोई अशुन्य भिन्न–भिन्न वास्तविक संख्याए इस प्रकार है कि $(a^2 + b^2 + c^2)p^2 - 2(ab + bc + cd)p + (b^2 + c^2 + d^2) = 0$ है, तब

(1) a, c, p गुणोतर श्रेणी में है (2) a, b, c, d गुणोतर श्रेणी में है
(3) a, b, c, d समान्तर श्रेणी में है (4) a, c, p समान्तर श्रेणी में है

Sol. **2**

$(a^2 + b^2 + c^2)p^2 - 2(ab + bc + cd)p + (b^2 + c^2 + d^2) = 0$
$(a^2p^2 - 2abp + b^2) + [b^2p^2 - 2bcp + c^2] + [c^2p^2 - 2cdp + d^2] = 0$
$(ap - b)^2 + (bp - c)^2 + (cp - d)^2 = 0$

$ap = b$ $\qquad \frac{b}{a} = \frac{c}{b} = \frac{d}{c} = p$
$bp = c$
$cp = d$ $\qquad$ a, b, c, d are in G.P.

Q.8 Two families with three members each and one family with four members are to be seated in a row. In how many ways can they be seated so that the same family members are not separated?

तीन सदस्यों वाले प्रत्येक दो परिवार तथा चार सदस्यों वाले एक परिवार को एक पंक्ति में बिठाया जाना है उन्हें कितने तरिको से बिठाया जा सकता है ताकि एक ही परिवार के सदस्य अलग ना हो ?

(1) 2! 3! 4! (2) $(3!)^3 \cdot (4!)$ (3) $3!\,(4!)^3$ (4) $(3!)^2 \cdot (4!)$

Sol. **2**

$F_1 \to 3$ members
$F_2 \to 3$ members
$F_3 \to 4$ members
No. of ways can they be seated so that the same family members are not separated
$= 3! \times 3! \times 3! \times 4! = (3!)^3 .\, 4!$

Q.9 The values of λ and μ for which the system of linear equations

$x + y + z = 2$
$x + 2y + 3z = 5$
$x + 3y + \lambda z = \mu$

has infinitely many solutions are, respectively:

(1) 6 and 8 (2) 5 and 8 (3) 5 and 7 (4) 4 and 9

λ तथा μ के मान, जिसके लिये रेखिय समीकरण निकाय

$x + y + z = 2$
$x + 2y + 3z = 5$
$x + 3y + \lambda z = \mu$

अनन्त कई हल रखते है क्रमशः है–

(1) 6 तथा 8 (2) 5 तथा 8 (3) 5 तथा 7 (4) 4 तथा 9

Sol. **2**

$x + y + z = 2$

$x + 2y + 3z = 5$

$x + 3y + \lambda z = \mu$

has infinitely many solutions

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & \lambda \end{vmatrix} = 0$$

$R_2 \rightarrow R_2 - R_1$

$R_3 \rightarrow R_3 - R_1$

$$\begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 2 & \lambda - 1 \end{vmatrix} = 0$$

$(\lambda - 1 - 4) = 0$

$\Rightarrow \lambda = 5$

$$\Delta_3 = \begin{vmatrix} 1 & 1 & 2 \\ 1 & 2 & 5 \\ 1 & 3 & \mu \end{vmatrix} = 0$$

$R_2 \rightarrow R_2 - R_1$

$R_3 \rightarrow R_3 - R_1$

$$\begin{vmatrix} 1 & 1 & 2 \\ 0 & 1 & 3 \\ 0 & 2 & \mu - 2 \end{vmatrix} = 0$$

$(\mu - 2 - 6) = 0$

$\Rightarrow \mu = 8$

$\lambda = 5, \mu = 8$

Q.10 Let m and M be respectively the minimum and maximum values of

$$\begin{vmatrix} \cos^2 x & 1 + \sin^2 x & \sin 2x \\ 1 + \cos^2 x & \sin^2 x & \sin 2x \\ \cos^2 x & \sin^2 x & 1 + \sin 2x \end{vmatrix}$$

Then the ordered pair (m, M) is equal to:

माना m तथा M , $\begin{vmatrix} \cos^2 x & 1 + \sin^2 x & \sin 2x \\ 1 + \cos^2 x & \sin^2 x & \sin 2x \\ \cos^2 x & \sin^2 x & 1 + \sin 2x \end{vmatrix}$

के क्रमशः न्यूनतम तथा अधिकतम मान है, तब क्रमित युग्म (m, M) बराबर है–

(1) (–3, –1) (2) (–4, –1) (3) (1, 3) (4) (–3, 3)

Sol. **1**

$$\begin{vmatrix} \cos^2 x & 1+\sin^2 x & \sin 2x \\ 1+\cos^2 x & \sin^2 x & \sin 2x \\ \cos^2 x & \sin^2 x & 1+\sin 2x \end{vmatrix}$$

$R_1 \to R_1 - R_2$, $R_3 \to R_3 - R_2$

$$\begin{vmatrix} -1 & 1 & 0 \\ 1+\cos^2 x & \sin^2 x & \sin 2x \\ -1 & 0 & 1 \end{vmatrix}$$

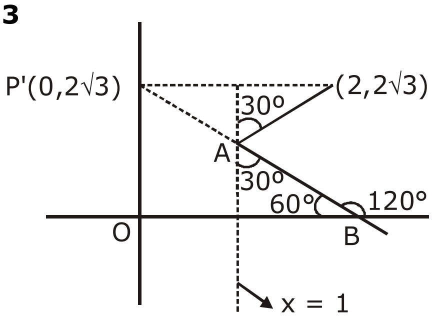
$\Rightarrow -1(\sin^2 x) -1(1 + \cos^2 x + \sin 2x)$
$\Rightarrow -\sin^2 x - \cos^2 x - 1 - \sin 2x$
$= -2 - \sin 2x$
$\therefore$ minimum value when $\sin 2x = 1$
$m = -2-1 = -3$
$\therefore$ Maximum value when $\sin 2x = -1$
$(m, M) = (-3, -1)$

Q.11 A ray of light coming from the point $(2, 2\sqrt{3})$ is incident at an angle 30º on the line x = 1 at the point A. The ray gets reflected on the line x = 1 and meets x-axis at the point B. Then, the line AB passes through the point:

बिन्दु $(2, 2\sqrt{3})$ से आती हुई एक प्रकाश की किरण बिन्दु A पर x = 1 रेखा पर एक 30° के कोण पर आपतित होती है, किरण रेखा x = 1 पर परावर्तित होती है तथा बिन्दु B पर x-अक्ष पर मिलती है तब रेखा AB निम्न बिन्दु से गुजरती है–

(1) $(4, -\sqrt{3})$ (2) $\left(3, -\frac{1}{\sqrt{3}}\right)$ (3) $(3, -\sqrt{3})$ (4) $\left(4, -\frac{\sqrt{3}}{2}\right)$

Sol. **3**



Equation of P'B $\to y - 2\sqrt{3} = \tan 120° (x - 0)$

$\sqrt{3}x + y = 2\sqrt{3}$

$(3, -\sqrt{3})$ satisfy the line

Q.12 Out of 11 consecutive natural numbers if three numbers are selected at random (without repetition), then the probability that they are in A.P. with positive common difference, is:
11 क्रमागत प्राकत संख्याओं मे से यदि तीन संख्याये यादच्छिक रूप से चुनी जाती है (बिना पुनरावती के) तब वह प्रायिकता कि वे धनात्मक सार्व अंतर के साथ समान्तर श्रेणी मे है, होगी–

(1) $\frac{10}{99}$ (2) $\frac{5}{33}$ (3) $\frac{15}{101}$ (4) $\frac{5}{101}$

Sol. **2**

Case-1
E, O, E, O, E, O, E, O, E, O, E
$2b = a + c \rightarrow$ Even
$\Rightarrow$ Both a and c should be either even or odd.

$$P = \frac{{}^{6}C_2 + {}^{5}C_2}{{}^{11}C_3} = \frac{5}{33}$$

Case -2
O, E, O, E, O, E, O, E, O, E, O

$$P = \frac{{}^{5}C_2 + {}^{6}C_2}{{}^{11}C_3} = \frac{5}{33}$$

Total probability = $\frac{1}{2} \times \frac{5}{33} + \frac{1}{2} \times \frac{5}{33} = \frac{5}{33}$

Q.13 If $f(x + y) = f(x) f(y)$ and $\sum_{x=1}^{\infty} f(x) = 2$, x, y$\in$N, where N is the set of all natural number, then the value of $\frac{f(4)}{f(2)}$ is :

यदि $f(x + y) = f(x) f(y)$ तथा $\sum_{x=1}^{\infty} f(x) = 2$, x, y$\in$N है, जहां N सभी प्राकत संख्याओं का समुल्य है, तब $\frac{f(4)}{f(2)}$ का मान है–

(1) $\frac{2}{3}$ (2) $\frac{1}{9}$ (3) $\frac{1}{3}$ (4) $\frac{4}{9}$

Sol. **4**

$f(x + y) = f(x) f(y)$
* Put x = 1, y = 1
$f(2) = (f(1))^2$
* Put x = 2, y = 1
$f(3) = f(2).\ f(1) = f((1))^3$
* Put x = 2, y = 2
$f(4) = f((2))^2 = f((1))^4$
$f(n) = (f(1))^n$

$\sum_{x=1}^{\infty} f(x) = f(1) + f(2) + f(3) + ... = 2$

$\Rightarrow f(1) + f((1))^2 + f((1))^3 .... = 2$

$\frac{f(1)}{1 - f(1)} = 2$

$f(1) = 2/3$

$f(2) = \left(\frac{2}{3}\right)^2, f(4) = \left(\frac{2}{3}\right)^4$

$\frac{f(4)}{f(2)} = \frac{(2/3)^4}{(2/3)^2} = \frac{4}{9}$

Q.14 If {p} denotes the fractional part of the number p, then $\left\{\frac{3^{200}}{8}\right\}$, is equal to :

यदि {p} , संख्या p के भिन्नात्मक भाग को प्रदर्शित करता है, तब $\left\{\frac{3^{200}}{8}\right\}$ बराबर है–

(1) $\frac{5}{8}$ (2) $\frac{1}{8}$ (3) $\frac{7}{8}$ (4) $\frac{3}{8}$

Sol. **2**

$\left\{\frac{3^{200}}{8}\right\} = \left\{\frac{9^{100}}{8}\right\} = \left\{\frac{(8+1)^{100}}{8}\right\}$

$\left\{\frac{{}^{100}C_0 1^{100} + {}^{100}C_1 (8) 1^{99} + {}^{100}C_2 (8^2) 1^{98} + ... + {}^{100}C_{100} 8^{100}}{8}\right\}$

$= \left\{\frac{{}^{100}C_0 1^{100} + 8k}{8}\right\}$

$= \left\{\frac{1 + 8k}{8}\right\} = \left\{\frac{1}{8} + k\right\} K \in I$

$= \frac{1}{8}$

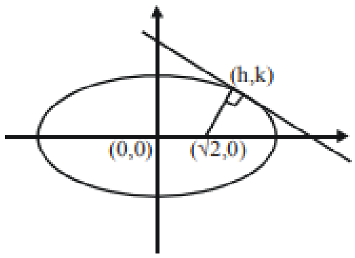
Q.15 Which of the following points lies on the locus of the foot of perpedicular drawn upon any tangent to the ellipse, $\frac{x^2}{4}+\frac{y^2}{2}=1$ from any of its foci ?

दीर्घवत $\frac{x^2}{4}+\frac{y^2}{2}=1$ पर इसकी किसी भी नाभियों से इसकी किसी स्पर्श रेखा के उपर खीचे गये लम्ब के पाद के बिन्दु पर स्थित निम्न बिन्दु है, होगा।

(1) $(-1, \sqrt{3})$ (2) $(-2, \sqrt{3})$ (3) $(-1, \sqrt{2})$ (4) $(1, 2)$

Sol. **4**

Let foot of perpendicular is (h,k)



$\frac{x^2}{4}+\frac{y^2}{2}=1$ (Given $)$

$a=2, b=\sqrt{2}, e=\sqrt{1-\frac{2}{4}}=\frac{1}{\sqrt{2}}$

$\therefore$ Focus $(ae, 0)=(\sqrt{2}, 0)$

Equation of tangent

$y=mx+\sqrt{a^2m^2+b^2}$

$y=mx+\sqrt{4m^2+2}$

Passes throguh (h,k) $(k-mh)^2=4m^2+2$

line perpendicular to tangent will have slope

$-\frac{1}{m}$

$y-0=-\frac{1}{m}(x-\sqrt{2})$

$my=-x+\sqrt{2}$

$(h+mk)^2=2$

Add equaiton (1) and (2) $k^2(1+m^2)+h^2(1+m^2)=4(1+m^2)$

$h^2+k^2=4$

$x^2+y^2=4$ (Auxilary circle)

$\therefore (-1,\sqrt{3})$ lies on the locus.

Q.16 $\lim_{x\to1}\left(\dfrac{\int_0^{(x-1)^2} t\cos(t^2)dt}{(x-1)\sin(x-1)}\right)$

(1) is equal to 1 (2) is equal to $\frac{1}{2}$ (3) does not xist (4) is equal to $-\frac{1}{2}$

$\lim_{x\to1}\left(\dfrac{\int_0^{(x-1)^2} t\cos(t^2)dt}{(x-1)\sin(x-1)}\right)$

(1) 1 के बराबर है (2) $\frac{1}{2}$ के बराबर है (3) विद्यमान नही है (4) $-\frac{1}{2}$ के बराबर है

Sol **Bouns**

$\lim_{x\to1}\left(\dfrac{\int_0^{(x-1)^2} t\cos(t^2)dt}{(x-1)\sin(x-1)}\right)$

Using L-Hopital rule

$$=\lim_{x\to1}\frac{2(x-1)\cdot(x-1)^2\cos(x-1)^4-0}{(x-1)\cdot\cos(x-1)+\sin(x-1)}\left(\frac{0}{0}\right)$$

$$=\lim_{x\to1}\frac{2(x-1)^3\cdot\cos(x-1)^4}{(x-1)\left[\cos(x-1)+\dfrac{\sin(x-1)}{(x-1)}\right]}$$

$$=\lim_{x\to1}\frac{2(x-1)^2\cos(x-1)^4}{(x-1)\left[\cos(x-1)+\dfrac{\sin(x-1)}{(x-1)}\right]}$$

$$=\lim_{x\to1}\frac{2(x-1)^2\cos(x-1)^4}{\cos(x-1)+\dfrac{\sin(x-1)}{(x-1)}}$$

on taking limit

$$= \frac{0}{1+1} = 0$$

Q.17 If $\sum_{i=1}^{n}(x_i - a) = n$ and $\sum_{i=1}^{n}(x_i - a)^2 = na$, (n, a > 1) then the standard deviation of n observations $x_1, x_2, \ldots, x_n$ is :

यदि $\sum_{i=1}^{n}(x_i - a) = n$ तथा $\sum_{i=1}^{n}(x_i - a)^2 = na$, (n, a > 1) है, तब n प्रेक्षणों $x_1, x_2, \ldots, x_n$ का मानक विचलन है–

(1) $n\sqrt{a-1}$ (2) $\sqrt{na-1}$ (3) a – 1 (4) $\sqrt{a-1}$

Sol. **4**

$$\text{S.D.} = \sqrt{\frac{\Sigma(x_i - a)^2}{n} - \left(\frac{\Sigma(x_i - a)}{n}\right)^2}$$

$$= \sqrt{\left(\frac{na}{n}\right) - \left(\frac{n}{n}\right)^2} = \sqrt{a-1}$$

Q.18 If α and β be two roots of the equation $x^2 - 64x + 256 = 0$. Then the value of $\left(\frac{\alpha^3}{\beta^5}\right)^{1/8} + \left(\frac{\beta^3}{\alpha^5}\right)^{1/8}$ is :

यदि α तथा β समीकरण $x^2 - 64x + 256 = 0$ के दो मूल है। तब $\left(\frac{\alpha^3}{\beta^5}\right)^{1/8} + \left(\frac{\beta^3}{\alpha^5}\right)^{1/8}$ का मान है–

(1) 1 (2) 3 (3) 2 (4) 4

Sol. **3**

$x^2 - 64x + 256 = 0$

$\alpha + \beta = 64$

$\alpha\beta = 256$

$$\left(\frac{\alpha^3}{\beta^5}\right)^{1/8} + \left(\frac{\beta^3}{\alpha^5}\right)^{1/8}$$

$$= \frac{\alpha + \beta}{(\alpha\beta)^{5/8}} = \frac{64}{(256)^{5/8}} = \frac{64}{32} = 2$$

Q.19 The position of a moving car at time t is given by $f(t) = at^2 + bt + c, t > 0$, where a, b and c are real numbers greater than 1. Then the average speed of the car over the time interval $[t_1, t_2]$ is attained at the point :

समय t पर एक गतिमान कार की स्थिति $f(t) = at^2 + bt + c, t > 0$ द्वारा दी गई है जहां a, b तथा c , 1 से अधिक वास्तविक संख्याऐं है। तब समय अंतराल $[t_1, t_2]$ पर कार की औसत चाल निम्न बिन्दू पर मिलती है, होगा–

(1) $(t_1 + t_2)/2$ (2) $2a(t_1 + t_2) + b$ (3) $(t_2 - t_1)/2$ (4) $a(t_2 - t_1) + b$

Sol. **1**

$$f'(t) = V_{av} = \frac{f(t_2) - f(t_1)}{t_2 - t_1}$$

$$= \frac{a(t_2^2 - t_1^2) + b(t_2 - t_1)}{t_2 - t_1}$$

$$= a(t_1 + t_2) + b = 2at + b$$

$$t = \frac{t_1 + t_2}{2}$$

Q.20 If $I_1 = \int_0^1 (1 - x^{50})^{100}\,dx$ and $I_2 = \int_0^1 (1 - x^{50})^{101}\,dx$ such that $I_2 = \alpha I_1$ then α equals to :

यदि $I_1 = \int_0^1 (1 - x^{50})^{100}\,dx$ तथा $I_2 = \int_0^1 (1 - x^{50})^{101}\,dx$ इस प्रकार है कि $I_2 = \alpha I_1$ है तब α बराबर है–

(1) $\frac{5050}{5049}$ (2) $\frac{5050}{5051}$ (3) $\frac{5051}{5050}$ (4) $\frac{5049}{5050}$

Sol. **2**

$$I_1 = \int_0^1 (1 - x^{50})^{100}\,dx$$

$$I_2 = \int_0^1 (1 - x^{50})(1 - x^{50})^{100}\,dx$$

$$= \int_0^1 (1 - x^{50})^{100}\,dx - \int_0^1 x^{50}(1 - x^{50})^{100}\,dx$$

$$I_2 = I_1 - \int_0^1 \underset{I}{x} - \underbrace{x^{49}(1 - x^{50})^{100}}_{II}\,dx$$

By using by parts

$1 - x^{50} = t$

$$\Rightarrow x^{49}\,dx = \frac{-dt}{50}$$

$$I_2 = I_1 - \left[x\left(\frac{-1}{50}\right)\frac{(1 - x^{50})^{101}}{101}\right]_0^1 + \int_0^1 \left(\frac{-1}{50}\right)\frac{(1 - x^{50})^{101}}{101}\,dx$$

$$I_2 = I_1 - 0 + \frac{\int_0^1 (1 - x^{50})^{101}}{(-5050)}\,dx$$

$$I_2 = I_1 - \frac{I_2}{5050}$$

$$\frac{5051}{5050} I_2 = I_1$$

$$I_2 = \frac{5050}{5051} I_1$$

$$\alpha = \frac{5050}{5051}$$

Q.21 If $\vec{a}$ and $\vec{b}$ are unit vectors, then the greatest value of $\sqrt{3}\,|\vec{a}+\vec{b}|+|\vec{a}-\vec{b}|$ is_____.

यदि $\vec{a}$, तथा $\vec{b}$ इकाई सदिश है तब $\sqrt{3}\,|\vec{a}+\vec{b}|+|\vec{a}-\vec{b}|$ का अधिकतम मान होगा

Sol. **4**

$$\sqrt{3}\,|\vec{a}+\vec{b}|+|\vec{a}-\vec{b}|$$

$$=\sqrt{3}(\sqrt{2+2\cos\theta})+\sqrt{2-2\cos\theta}$$

$$=\sqrt{6}(\sqrt{1+\cos\theta})+\sqrt{2}(\sqrt{1-\cos\theta})$$

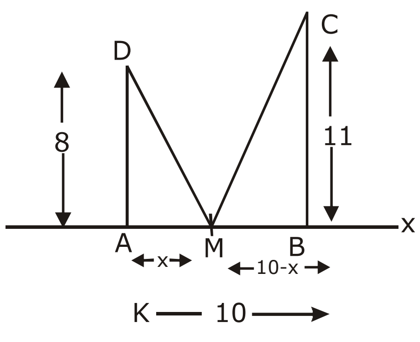
$$=2\sqrt{3}\left|\cos\frac{\theta}{2}\right|+2\left|\sin\frac{\theta}{2}\right|$$

$$\le\sqrt{(2\sqrt{3})^2+(2)^2}=4$$

Q.22 Let AD and BC be two vertical poles at A and B respectively on a horizontal ground. If AD = 8 m, BC = 11 m and AB = 10 m; then the distance (in meters) of a point M on AB from the point A such that $MD^2 + MC^2$ is minimum is _______.

माना AD तथा BC एक क्षेतिज धरातल पर क्रमशः दो उर्ध्वाधर खम्भे A तथा B है। AD = 8 m, BC = 11 m तथा AB = 10 m है तब बिन्दु A से AB पर एक बिन्दु m की दूरी (मीटर में) इस प्रकार है कि $MD^2 + MC^2$ न्यूनतम है, होगी–

Sol. **5**



$(MD)^2 = x^2 + 8^2 = x^2 + 64$
$(MC)^2 = (10-x)^2 + (11)^2 = (x-10)^2 + 121$
$f(x) = (MD)^2 + (MC)^2 = x^2 + 64 + (x-10)^2 + (121)$
Differentiate
$f'(x) = 0$
$2x + 2(x-10) = 0$
$4x = 20 \Rightarrow x = 5$
$f''(x) = 4 > 0$
at $x = 5$ point of minima

Q.23 Let $f : R \to R$ be defined as

$$f(x) = \begin{cases} x^5 \sin\left(\frac{1}{x}\right) + 5x^2, & x < 0 \\ 0, & x = 0 \\ x^5 \cos\left(\frac{1}{x}\right) + \lambda x^2, & x > 0 \end{cases}$$

The value of λ for which $f''(0)$ exists, is ________.
माना $f : R \to R$,

$$f(x) = \begin{cases} x^5 \sin\left(\frac{1}{x}\right) + 5x^2, & x < 0 \\ 0, & x = 0 \\ x^5 \cos\left(\frac{1}{x}\right) + \lambda x^2, & x > 0 \end{cases}$$

के रूप मे परिभाषित है, तब λ का मान जिसके लिये $f''(0)$ विद्यमान है, होगा–

Sol. **5**

$$f(x) = \begin{cases} x^5 \sin\left(\frac{1}{x}\right) + 5x^2, & x < 0 \\ 0, & x = 0 \\ x^5 \cos\left(\frac{1}{x}\right) + \lambda x^2, & x > 0 \end{cases}$$

$$f'(x) \begin{cases} 5x^4 \sin\left(\frac{1}{x}\right) - x^3 \cos\left(\frac{1}{x}\right) + 10x, & x < 0 \\ 0, & x = 0 \\ 5x^4 \cos\left(\frac{1}{x}\right) + x^3 \sin\left(\frac{1}{x}\right) + 2\lambda x, & x > 0 \end{cases}$$

$$f''(x) = \begin{cases} 20x^3 \sin\left(\frac{1}{x}\right) - 5x^2 \cos\left(\frac{1}{x}\right) - 3x^2 \cos\left(\frac{1}{x}\right) - x \sin\left(\frac{1}{x}\right) + 10, & x < 0 \\ 0, & x = 0 \\ 20x^3 \cos\left(\frac{1}{x}\right) + 5x^2 \sin\left(\frac{1}{x}\right) + 3x^2 \sin\left(\frac{1}{x}\right) - x \cos\left(\frac{1}{x}\right) + 2\lambda, & x > 0 \end{cases}$$

$f''(0^+) = f''(0^-)$

$2\lambda = 10 \Rightarrow \lambda = 5$

Q.24 The angle of elevation of the top of a hill from a point on the horizontal plane passing through the foot of the hill is found to be 45°. After walking a distance of 80 meters towards the top, up a slope inclined at an angle of 30° to the horizontal plane, the angle of elevation of the top of the hill becomes 75°. Then the height of the hill (in meters) is _______.

पहाडी के पाद से गुजरते हुये एक क्षेतिज समतल पर एक बिन्दू से एक पहाडी के शिखर का उनन्यन कोण 45° पाया जाता है। क्षेतिज तल से 30° के कोण पर झुके ढाल के उपर शिखर की ओर 80 मीटर की दूरी चलने के पश्चात, पहाडी के शिखर का उन्नयन कोण 75° हो जाता है। तब पहाडी की ऊचाई (मीटर में) होगी–

Sol. **80**

$x = 80 \cos 30° = 40\sqrt{3}$

$y = 80 \sin 30° = 40$

In ΔADC

$\tan 45° = \frac{h}{x+z} \Rightarrow h = x + z$

$\Rightarrow h = 40\sqrt{3} + z$ ....(i)

In ΔEDF

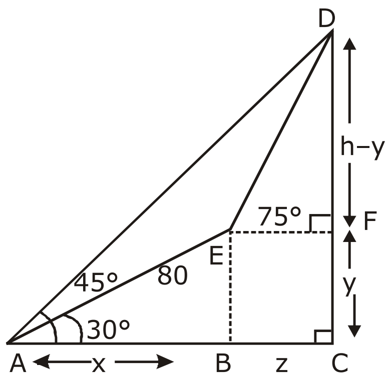
$\tan 75° \; \frac{h-y}{z}$

$2 + \sqrt{3} = \frac{h-40}{z} \Rightarrow z = \frac{h-40}{2+\sqrt{3}}$ .....(ii)

Put the value of z from (i)

$h - 40\sqrt{3} = \frac{h-40}{2+\sqrt{3}}$

x =



$h(1 + \sqrt{3}) = 40(2\sqrt{3} + 3 - 1)$

$h(1 + \sqrt{3}) = 80(1 + \sqrt{3})$

$h = 80$

Q.25 Set A has m elements and set B has n elements. If the total number of subsets of A is 112 more than the total number of subsets of B, then the value of m.n is _______.

समुच्चय A, m अवयव रखते है तथा समुच्च B , n अवयव रखते है। यदि A के उपसमुच्यों की कुल संख्या 112 है जो B के उपसमुच्यों की कुल संख्या से अधिक है, तब M. N मान है–

Sol. **28**

A & B are two set

No. of subsets of A = 2^m

No. of subsets of B = 2^n

$2^m = 2^n + 112$

$2^m - 2^n = 112$

$2^n(2^{m-n}-1) = 112$

$2^n(2^{m-n}-1) = 2^4(2^3-1)$

$n = 4$ $\qquad$ $m - n = 3$

$m - 4 = 3 \Rightarrow m = 7$

$m.\ n = 28$

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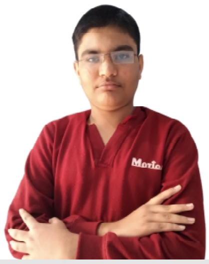

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